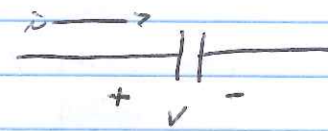
3/24/2015 Chapter 7 - Inductors + Capacitors

- Capacitors C + Inductors L
- L's + C's can store + deliver infinite amounts of energy.

Move into the realm of active vs. passive elements.
Active elements supply energy and can supply an average power greater than Ø over an infinite time interval

$\boxed{\text{Capacitance}}$ C is defined by a V-I relationship
where $i = C\frac{dv}{dt}$ V and I are functions of time!
Capacitance is an ampere-second per volt, or coulomb per volt which is equivalent to one Farad (F) $\frac{\text{Coulomb}}{\text{Volt}}$

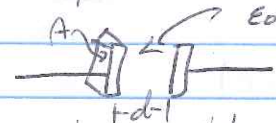
$i = C\frac{dv}{dt} \Rightarrow dv = \frac{i(t)\,dt}{C}$



$i = \frac{dq}{dt}$ change in charge on plate over time

$\varepsilon_0 = 8.85\,pF/m$ max

$C = \frac{\varepsilon A}{d}$



two parallel conducting plates
plates have area A separated by insulating conductor w/ ε = permittivity
separated by distance d.

C of several hundred μF is very large

$\boxed{\text{Example 7.1}}$ $i = C\frac{dv}{dt}$ a) $i = 2 \times 5\frac{dv}{dt} = 0A$

b) First waveform is a sine, derivative is a cosine, multiplied by 2 w/ same frequency.

$\boxed{\text{Practice 7.1}}$

Determine I if $V = 2e^{-5t}\,V$ w/ $C = 5mF$

$$I = C\frac{dv}{dt} = 5\times10^{-3} \times 2e^{-5t}\frac{d}{dt} = -5\times5\times10^{-3}\times2e^{-5t}$$
$$= -50e^{-5t}\,mA = i(t)$$

$i(t) = C\frac{dv}{dt} \Rightarrow \partial v(t) = \frac{1}{C} i(t)\, \partial t$

Integrate both sides between t_0 + t

we get ~~$v(t) = \frac{1}{C}\int_{t_0}^{t} i(t)$~~

$$\int_{v(t_0)}^{v(t)} 1\,dv = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt'$$

$$= v(t) - v(t_0) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt'$$

$$\therefore v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0) \quad \text{or } v(t) = \frac{1}{C}\int i\,dt + K$$

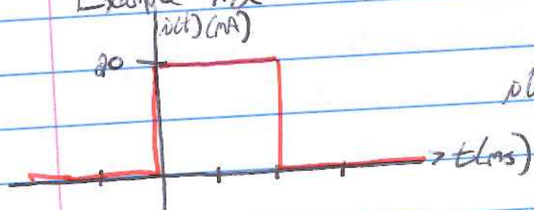
in many cases $v(t_0)$ is not discernable + usually

~~$v(t_0)$~~ $t_0 = -\infty$ + $v(-\infty) = 0$

such that $v(t) = \frac{1}{C}\int_{-\infty}^{t} i\,dt'$

Again $q(t) = Cv(t)$

Example 7.2



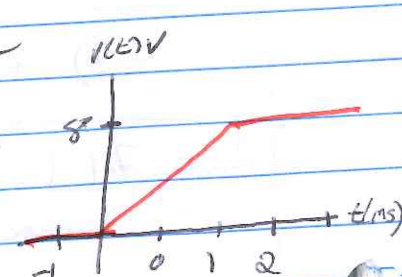
solve over intervals

$$i(t) = \begin{cases} 0 & -\infty \le t < 0 \\ 20\text{mA} & 0 \le t < 2\text{ms} \\ 0 & 2\text{ms} \le t < \infty \end{cases}$$

Interval (1) $i(t_0) =$ ~~...~~ $= 0 \therefore v(t_0) = 0$

$$v(t) \begin{cases} 0 & -\infty \le t < 0 \\ 4000t & 0 \le t \le 2\text{ms} \end{cases}$$

(2) $v(t) = \frac{1}{C}\int_{t_0}^{t} i(t')\,dt' + v(t_0)$



Because $i(t_0) = 0$ $v(t_0) = 0$

$$\frac{1}{5\mu F}\int_{0}^{2ms} 20^{mA}\,dt = \frac{1}{5 \times 10^{-6}} \times 20 \times 10^{-3} \cdot t' \Big|_0^t$$

$= v(t) = 4000\,t \quad 0 \le t \le 2\text{ms}$

Interval (3) $v(t_0) = v(2\text{ms}) = 4000 \times 2\text{ms} = 8V \quad V \ge 2\text{ms} = 0$

$\therefore v(t)\ 2\text{ms} \le t \le \infty = 8V + 0 = 8V$

Energy Storage $\quad p = v \cdot i = Cv\frac{dv}{dt} \Rightarrow p\,dt = Cv\,dv$

Change in Energy over time: $\displaystyle\int_{t_0}^{t} p\,dt' = C\int_{v(t_0)}^{v(t)} v'\,dv'$

$p = \frac{\partial E}{\partial t}$ $\quad \int p = \text{Energy}$

$$= \left[\frac{v(t)^2}{2} - \frac{v(t_0)^2}{2}\right] \times C$$

$w_c(t) - w_c(t_0) = \frac{C}{2}\left\{[v(t)]^2 - [v(t_0)]^2\right\}$

if Energy at $t_0 = 0$ then we have

$$w_c(t) = \frac{1}{2}Cv^2$$

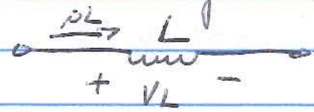
$\boxed{\text{Ideal Capacitor}}$

- no current if voltage is not changing with time. Open circuit to DC
- Finite amount of energy can be stored across the capacitor (current zero, voltage constant)
- Capacitor cannot change voltage instantaneously, but current through can
- Stores ~~energy~~ energy does not dissipate energy.

$\boxed{\text{Inductor}}$

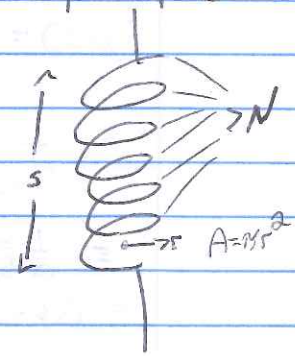
L is also defined by a voltage - current relationship

$v = L\frac{di}{dt}$



Henry (H)
volt-second per ampere

Inductor is modelled by winding a wire into a coil.
where inductance is determined by: $\mu N^2 A / s$
N: # of complete turns of wire
A: is cross section area
s: is the axial length of the helix
μ: is material permittivity. $\mu_0 = 4\pi \times 10^{-7}\,H/m$



voltage across an inductor is related to time rate of change of the current through it

no voltage across an inductor yields a short circuit ~~~~ when current is constant

Instantaneous changes in voltage is possible, but NOT current through an inductor

Example 7.4 $v = L\frac{di}{dt}$

$\therefore \quad di = \frac{1}{L} v\,dt$

$$\int_{i(t_0)}^{i(t)} di' = \frac{1}{L}\int_{t_0}^{t} v(t')\,dt'$$

$$i(t) - i(t_0) = \frac{1}{L} \cancel{v(t_0)\,t(t_0)} \int_{t_0}^{t} v\,dt'$$

$$\hookrightarrow \quad i(t) = \frac{1}{L}\int_{t_0}^{t} v\,dt' + i(t_0)$$

$$\therefore \; i(t) = \frac{1}{L}\int v\,dt + k$$

if $i(t_0) = i(-\infty) = 0$, $\quad i(t) = \frac{1}{L}\int_{-\infty}^{t} v\,dt'$

$\boxed{\text{Example } 7.6}$ $v(t) = 6\cos 5t \; V$

$$i(t) = \frac{1}{L}\int_{t_0}^{t} 6\cos 5t\,dt + i(t_0)$$

$$= \frac{1}{2}\int_{t_0}^{t} 6\cos 5t\,dt + i(t_0)$$

$$= \frac{1}{2}\frac{6}{5}\sin 5t - \frac{1}{2}\frac{6}{5}\sin 5(t_0) + i(t_0)$$

$$= 0.6\sin 5t - 0.6\sin 5t_0 + i(t_0)$$

plug in $-\frac{\pi}{2}$ for $t_0 = \left(-\frac{\pi}{2}\right)$ $\quad i(t_0) = 1A$

$\therefore\ i(t) = 0.6 \sin 5t - 0.6 \sin 5 \cdot -\frac{\pi}{2} + 1$

$= 0.6 \sin 5t + 1.6 = i(t)$

Energy Storage $\quad p = VI = Li\frac{di}{dt}$

$\Rightarrow p\,dt = Li\,di$

looking for energy w_L $\quad \displaystyle\int_{t_0}^{t} p\,dt' = L\int_{i(t_0)}^{i(t)} i'\,di'$

$= w_L(t) - w_L(t_0) = \frac{1}{2}L\left[i(t)^2 - i(t_0)^2\right]$

$\therefore\ w_L(t) = \frac{L}{2}\left[i(t)^2 - i(t_0)^2\right] + w_L(t_0)$

if energy is 0 at time 0, $\boxed{w_L(t) = \frac{1}{2}Li^2}$

Ideal Inductor

- No voltage across inductor if current is not changing w/ time. (short-circuit to DC)
- Finite amount of energy ~~[illegible]~~ can be stored in inductor, even if voltage across is zero.
- Current cannot change instantaneously, voltage can.
- Inductors do not dissipate energy, they only store it.

Series + Parallel

1. L's in series act like R's in series
2. L's in parallel act like R's in parallel
3. C's in series act like R's in parallel
4. C's in parallel act like R's in series